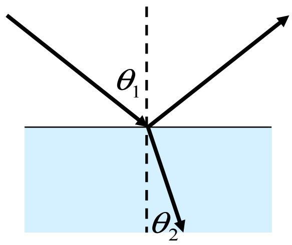
Geometrical optics

Lenses and Mirrors

- Optical devices are combination of lenses and mirrors
  - Telescopes, microscopes, cameras, binoculars …
- Wave-ness of light creates interference phenomena
  - Reflectivity depends on thickness d
    - e.g. anti-reflective coating $d/\lambda = 1/4$
  - Resolution limited by diffraction due to aperture a
    - Rayleigh diffraction limit $\theta > 1.22\lambda/a$
- Magnitude of interference depend on *l*/(size)
  - If size >> wavelength, we can ignore these effects
  - Most lenses satisfy this condition

Geometrical Optics

- Assume all elements (lenses/mirrors) are much larger than the wavelength in aperture and thickness
  - We can treat light as if it's a particle
  - Trajectory in each medium is a straight line
  - At boundaries, it either reflects or refracts
    - Refraction angle given by Snell's law

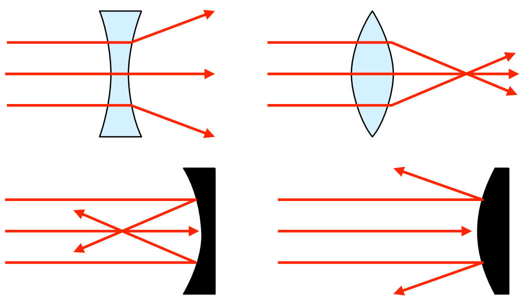


$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$$

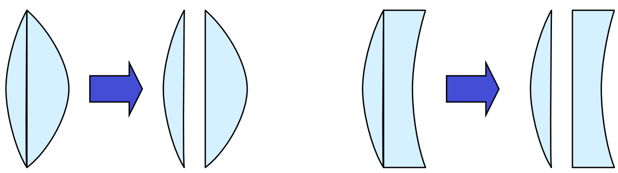
- Everything is determined by the elements' shapes, indices of refraction, and their geometrical arrangement
  - Geometrical Optics = Analysis of optical devices using this approximation

Optical Elements

- There are 4 major types:
  - Concave and convex lenses
  - Concave and convex mirrors

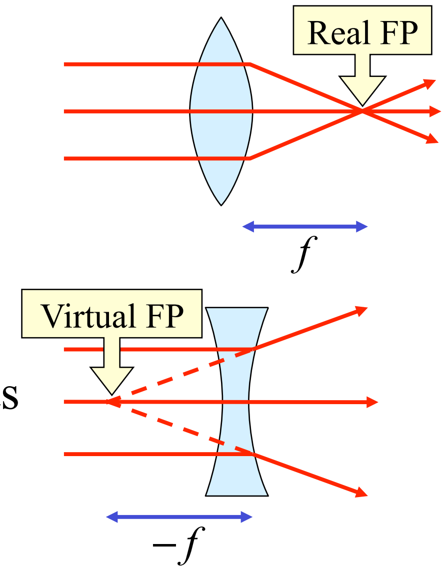


- Lenses may have different radii on two surfaces
  - Consider them as a combination of two lenses with one side flat



Focal Points

- Lenses (mirrors) turn plane waves into spherical waves
  - "Origin" of the spherical waves is the focal point
- Light may or may not actually go through the focal point
  - If yes → real focal point
  - If not → virtual focal point
- Distance between the lens and its focal point = focal length f
  - If real → $f > 0$
  - If virtual → $f < 0$ ← convention



Convex Lens

- Let's start with a flat-convex lens
  - Convex side is spherical
  - Snell's law

$$n \sin\theta_1 = \sin\theta_2$$

$$\sin\theta_2 = \frac{ny}{R}$$

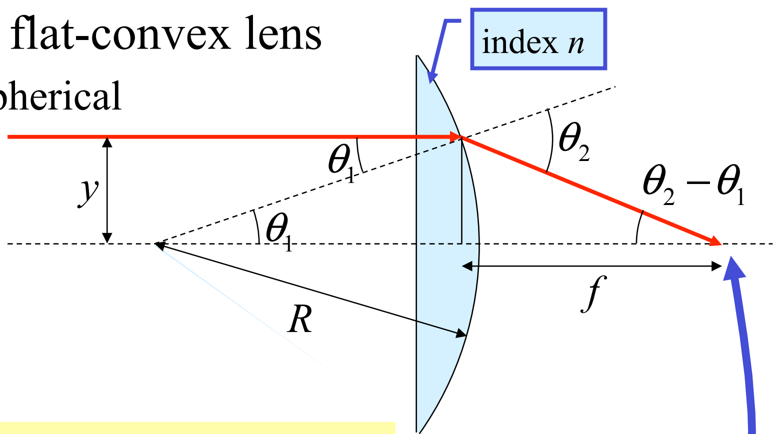
  - For small angles

$$f = \frac{y}{\tan(\theta_2 - \theta_1)} \approx \frac{y}{\theta_2 - \theta_1} \approx \frac{y}{\frac{ny}{R} - \frac{y}{R}} = \frac{R}{n-1}$$

← Focal length

- Incoming light converges at the focal point

But there are approximations



Concave Lens

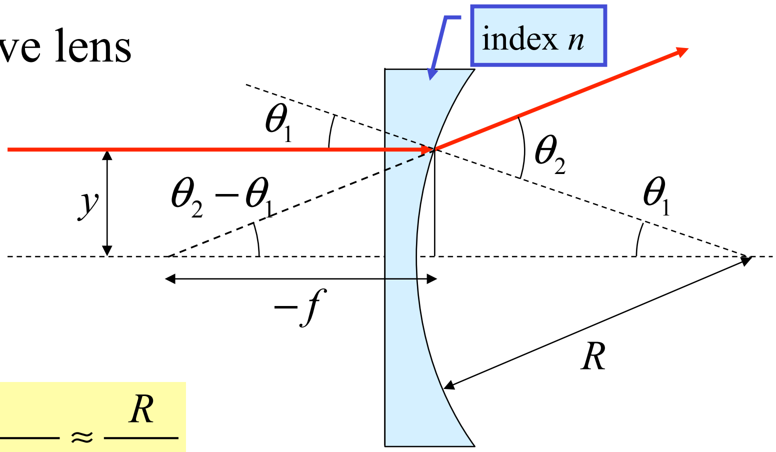
- Now a flat-concave lens
  - Snell's law

$$n \sin\theta_1 = \sin\theta_2$$

$$\sin\theta_2 = \frac{ny}{R}$$

  - For small angles

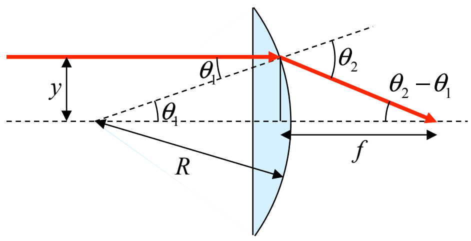
sign! $$-f = \frac{y}{\tan(\theta_2 - \theta_1)} \approx \frac{R}{n-1}$$



- Same formula, just a negative sign

Aberration

- Two approximations were made
  - Angles are small
  - Index n is a constant
- Both are incorrect for real lenses
  - Angles may get large if the aperture is large
    → Rays at different y do not converge at the same f
    - Spherical Aberration
  - Index varies with wavelength / due to dispersion
    → Rays with different / do not converge at the same f
    - Chromatic Aberration

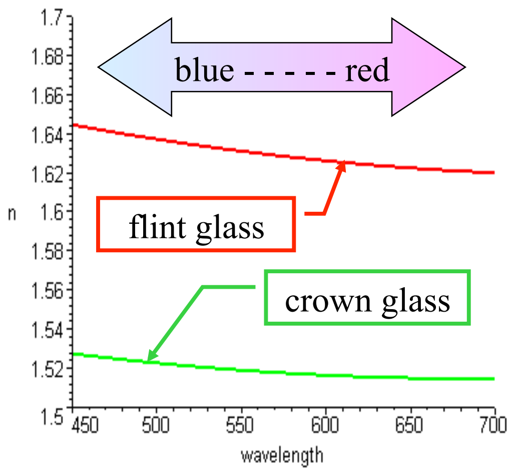


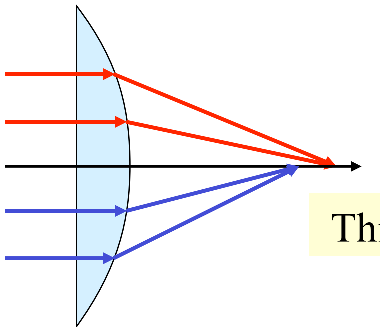
Spherical Aberration

- Spherical aberration can be reduced by
  - High-index glass (flint glass)
  - Aspherical (hyperbolic) lens shape
    - Not a perfect solution: doesn't work for off-axis light
    - Difficult to make with traditional polishing technique
  - Combining multiple lenses so that aberrations cancel
    - Mathematical technique known since 1830

Chromatic Aberration

- Index of refraction varies with wavelength $n_{\text{blue}} > n_{\text{red}}$
  - Blue light bends more than red light
  - Shorter *f* for blue than for red $f = \dfrac{R}{n-1}$

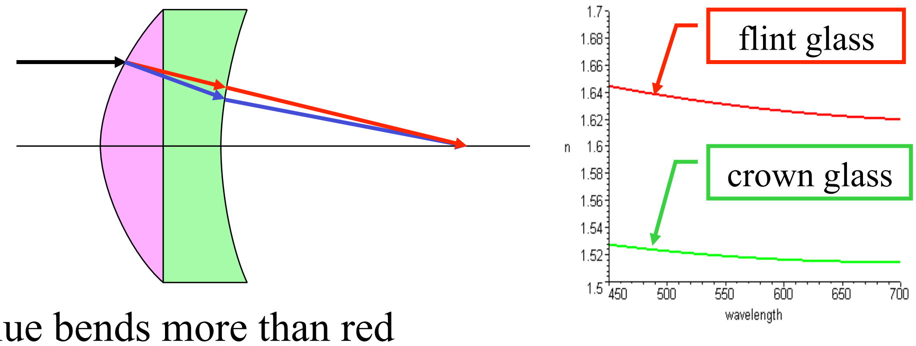




This is chromatic aberration

Achromatic Lenses

- How can we get rid of chromatic aberration?
  - Idea: use two kinds of glass with different dispersion



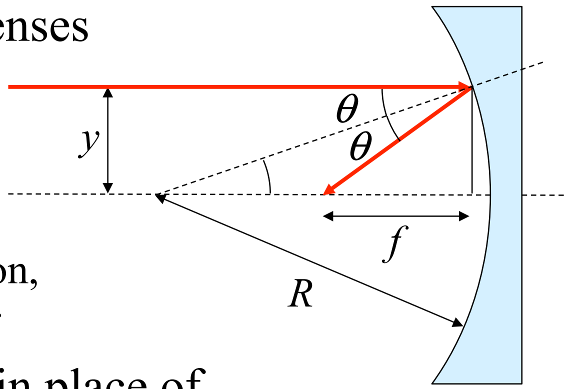
  - Blue bends more than red
    → Combine convex and concave lenses so that the dispersion cancels out → Achromatic lens

Mirrors

- Mirrors are simpler than lenses
  - For small angle

    $f = R/2$

  - No chromatic aberration
  - To avoid spherical aberration, you need a parabolic mirror

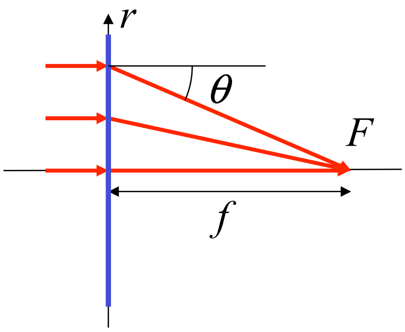


- Concave mirrors are used in place of convex lenses in telescopes
  - Easier to make a large mirror than a large lens
  - Can make the overall length shorter

Ideal Lens

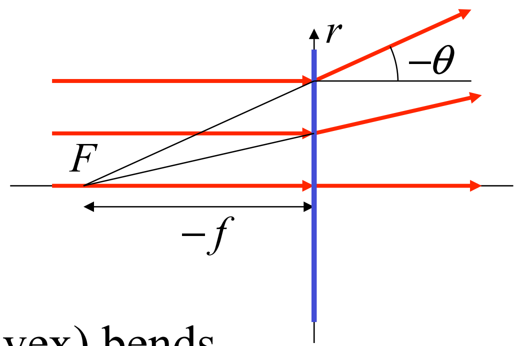
- An ideal lens would use very-high-index, non-dispersive material
  - Since $f = \frac{R}{n-1}$ such a lens have very large R
  - It can be made very thin, with no spherical aberration
- In the $n \to \infty$ limit, we find an infinitely thin film
  - Light entering the film magically bends by $\theta = \theta(r)$ that satisfies

$$\tan\theta = \frac{r}{f}$$



Ideal Lens

- What about a concave lens?
  - Easy: $\tan\theta = \frac{r}{f}$
    - Negative signs on θ and f cancel each other

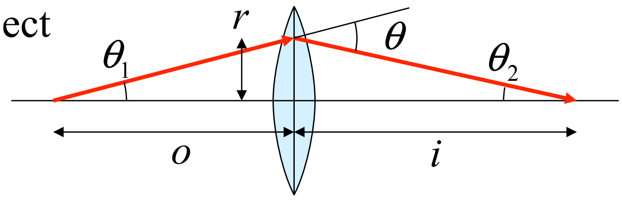


- An ideal lens (concave or convex) bends the light that passes at radius r by θ that satisfies

$$\tan\theta = \frac{r}{f}$$

Lens Formula

- First, we trace rays of light from a point through a lens
  - We assume ideal lens with no aberration
  - o = distance from the object
  - i = distance to the image



- Assuming small angles

$$\theta = \theta_1 + \theta_2 \approx \frac{r}{o} + \frac{r}{i}$$ ← geometry

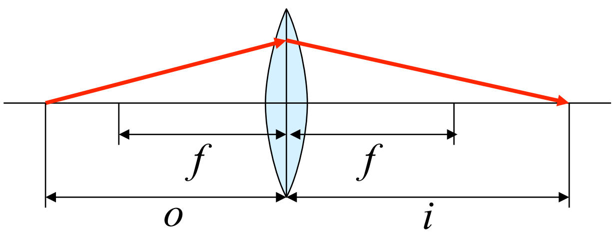
$$\theta \approx \tan\theta = \frac{r}{f}$$ ← ideal lens

➡ $$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$$ ← General "lens formula"

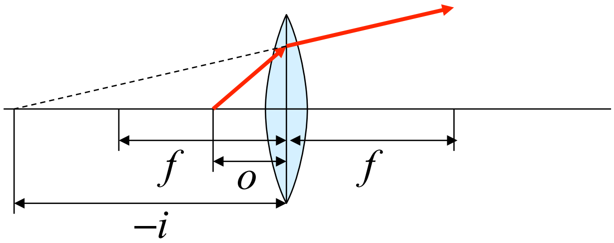
  - This is more useful that it looks…

Lens Formula $\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$

- The formula works in all combinations of o, i, and f



$o > f$ ➡ $i = \frac{of}{o-f} > 0$

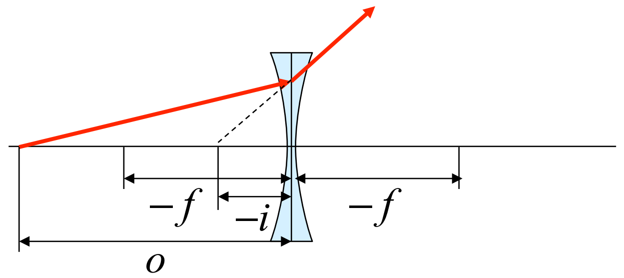


$o < f$ ➡ $i = \frac{of}{o-f} < 0$

  - Negative i → Image is virtual, i.e. light does not actually focus in a point

Lens Formula $\frac{1}{o}+\frac{1}{i}=\frac{1}{f}$

- It works with concave lenses as well

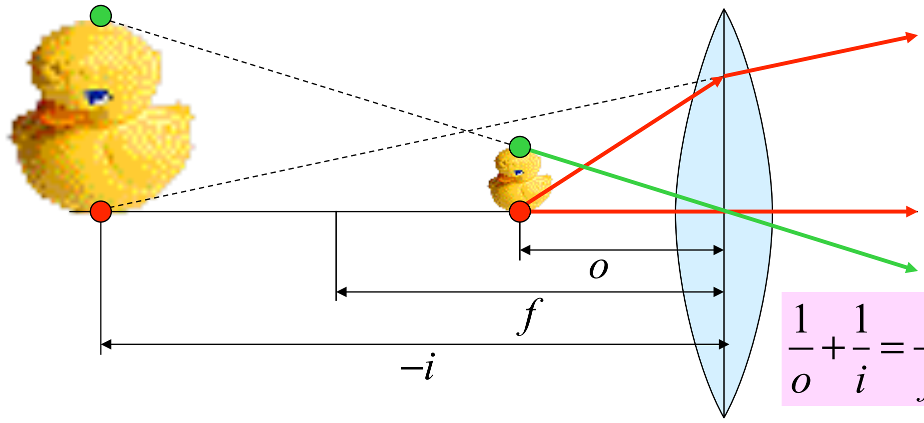


$f<0 \Rightarrow i=\frac{of}{o-f}<0$

  - Since f is negative, i is always negative
    → Image is virtual
- What do we mean by "images"?
  - So far our "object" is a point
  - How does a real object (with size) look through lenses?

Magnifying Glass

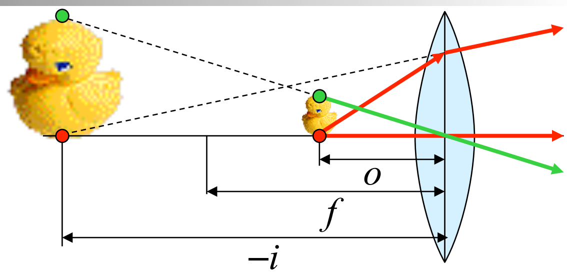
- Simplest optical device: a magnifying glass



$\frac{1}{o}+\frac{1}{i}=\frac{1}{f}$

  - We know image distance from the lens formula
  - Rays passing the middle of the lens don't bend
  - We can trace rays from various points of the object

Magnification



- Magnification power is

$$m = -\frac{i}{o}$$

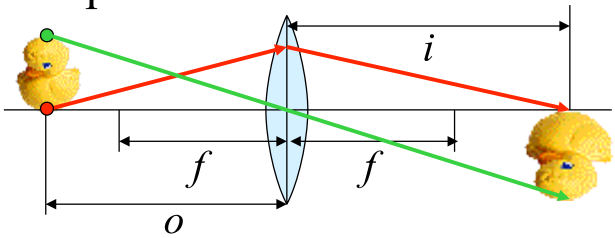
  - Using the lens formula

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f} \Rightarrow m = \frac{f}{f - o}$$

Object should be placed slightly inside the focal point

- If object is outside the focal point
  - Image is inverted
  - Magnification is again

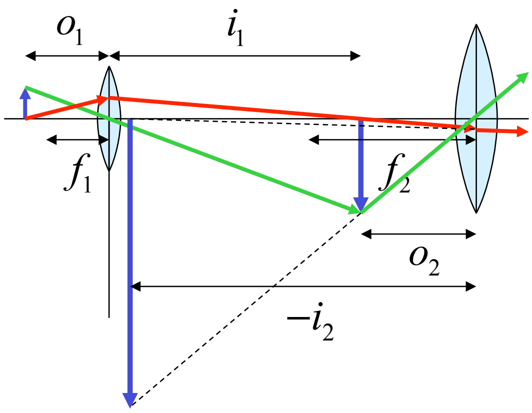
$$m = -\frac{i}{o} = \frac{f}{f - o}$$



Microscopes

- Combine the two cases to make a microscope
  - Object just outside f_1
  - Intermediate image is inverted and just inside f_2
  - Final image is inverted
  - Total magnification is

$$m = m_1 m_2 = \frac{i_1}{o_1} \frac{i_2}{o_2}$$



- This is getting tedious
  - We want a simpler mathematical formalism to deal with more complex optical devices

Summary

- Introduced geometrical optics
  - Size of optical elements much larger than the wavelength
    → Interference and diffraction can be ignored
- Discussed lenses
  - Focal point and focal length $$f = \frac{R}{n-1}$$
  - Real lenses are imperfect – aberrations
    - Spherical aberration when aperture is large
    - Chromatic aberration due to dispersion of the glass
    - There are solutions, but nothing is perfect
- Discussed ideal lens and lens formula
  - Ideal lens bends light by
  - Such a lens create an image at distance given by $\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$
  - Analyzed magnifying glass and microscope